Analytical Analysis of Propellers Using MATLAB

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Abstract *—A propeller is a device designed to convert rotational motion from an engine or motor into thrust, which in turn propels a vehicle. This study focuses on developing a computational tool to analyze the performance of various propeller geometries at a constant rotational speed across a range of advance ratios. Utilizing the Blade Element Momentum Theory (BEMT), this tool integrates MATLAB coding to simulate and compare performance metrics with empirical data from wind tunnel tests on the APC 10x7 Thin Electric propeller. This approach not only aids in understanding the aerodynamic characteristics of propellers but also enhances the design and optimization processes by providing a deeper insight into the influence of different geometric parameters on propeller performance.*

Key Terms *—Blade element momentum theory, Power coefficient, Propeller efficiency, Thrust coefficient.*

INTRODUCTION

When predicting the performance of a propeller based on aerodynamic analysis, it is crucial to consider several designs and analytical aspects. Propellers are vital components in various vehicles, from airplanes to drones, and their efficiency significantly impacts overall performance. The selection of the type of analysis to be performed on the propeller-air system is key. Often, the system is analyzed as an open system—commonly referred to as a control volume—due to the continuous flow of air across the propeller. This approach is pivotal to the application of momentum theory in analyzing thrust-generating devices.

Momentum theory, which forms the basis of our analysis, makes several assumptions [1], [2]:

- The fluid flow is homogeneous, incompressible, and in a steady state.
- There is no frictional drag.
- The propeller is considered a solid disk.
- The velocity is uniform, altering only in the flow direction.
- The wake after the rotor does not rotate.
- The static pressures far upstream and far downstream from the rotor are equal to the ambient static pressure.
- There is an increase in pressure across the rotating disk.

These assumptions are essential as they simplify the complex interactions within the system, making it possible to predict propeller performance more reliably. However, it's important to note that while momentum theory provides significant insights, it also introduces certain limitations. For instance, it only considers flow quantities at the system's inlet and outlet without accounting for detailed propeller blade characteristics like pitch angle, number of blades, and airfoil shape. Moreover, post-rotor fluid rotation induced by the blade's movement can reduce the amount of energy converted to useful work, a phenomenon known as wake rotation.

To address these challenges, our analysis incorporates both blade design and overall performance considerations using momentum theory along with the additional effects of wake rotation. The integration of blade element theory with momentum theory enables a comprehensive analysis that reflects both individual blade performance and the aggregate effects on the propeller system. By leveraging MATLAB, we have developed a computational tool that simulates and compares propeller performance metrics with empirical data, offering deeper insights into the aerodynamic characteristics of propellers.

The significance of this study lies in its potential to enhance the design and optimization processes of propellers. Understanding the influence of different geometric parameters on propeller performance can lead to more efficient and effective designs, applicable across various fields such as aerospace, marine, and wind energy sectors. This comprehensive analysis not only bridges the gap between theoretical assumptions and practical applications but also provides a robust framework for future propeller performance studies.

To achieve this, we developed an algorithm based on Blade Element Momentum Theory (BEMT) and implemented it using MATLAB. The next section outlines the algorithm's detailed steps and procedures.

ANALYSIS ALGORITHM FOR A PROPELLER USING BLADE ELEMENT MOMENTUM THEORY (BEMT)

To conduct a propeller performance analysis using Blade Element Momentum Theory (BEMT), we begin by establishing the flight conditions. This involves selecting the flight speed (U), determining the operational altitude, which affects air density, and setting the engine or motor revolutions per minute (RPM) to account for the propeller’s rotational speed.

Next, we detail the propeller's geometry. This starts with measuring the rotor diameter (D) and setting the blade pitch angle (θ_p), which defines the blades' orientation relative to the plane of rotation. We then determine the chord (c) at various radial positions (r) along each blade to assess their width distribution. Additionally, specifying the number of blades (B) on the propeller is crucial as it significantly influences both the generated thrust and the overall aerodynamic efficiency.

For a comprehensive analysis, we divide the propeller blade into N discrete segments, typically ranging from 10 to 20. The next critical step is obtaining the aerodynamic profiles of the propeller. This involves collecting data on the lift coefficient (Cl) and the drag coefficient (Cd) as functions of the angle of attack across varying Reynolds and Mach numbers. This aerodynamic data is crucial for depicting how the angle of attack, fluid viscosity (Reynolds numbers), and compressibility effects (Mach numbers) influence the interactions between the airfoil and the surrounding air, providing essential insights into the dynamic forces at play.

Once all the necessary data has been collected, the steps for the analysis procedure are as follows:

1. Initial Estimates:
   - Set initial estimates for the axial (a) and rotational (a') induction factors to begin the analysis.
2. Compute relative wind angle:
   - Calculate the angle of the relative wind, ϕi, using the formula:

$$\varphi_i = \tan^{-1}\left(\frac{(1+a)}{\lambda_{r,i}(1-a')}\right) \tag{1}$$

Where:

- Tip speed ratio:

$$\lambda = \left(\frac{R\Omega}{U}\right) \tag{2}$$

- Local tip speed ratio:

$$\lambda_{r,i} = \lambda\left(\frac{r_i}{R}\right) \tag{3}$$

3. Compute angle of attack using:

$$\alpha_i = \varphi_i - \theta_{p,i} \tag{4}$$

4. Obtain $C_l(\alpha)$ and $C_d(\alpha)$ using the angle of attack and the airfoil aerodynamic data.
5. Compute $C_n(\alpha)$ and $C_t(\alpha)$ using:

$$C_n = C_l \cos\varphi_i + C_d \sin\varphi_i$$

$$C_t = C_l \sin\varphi_i - C_d \cos\varphi_i \tag{5}$$

6. Update induction factors using:
   - Update the axial induction factor with Glauert correction:

$$a = \frac{1}{\left[4F\frac{\sin^2\varphi_i}{\sigma_i' C_n}+1\right]} \tag{6}$$

Where:

- The solidity is defined as:

$$\sigma_i' = \frac{Bc_i}{2\pi r_i} \tag{7}$$

- Prandtl’s Tip Loss Factor:

$$F_i = \frac{2}{\pi}\cos^{-1}\left[e^{-\left(\left\{\frac{\left(\frac{B}{2}\right)\left[1-\left(\frac{r_i}{R}\right)\right]}{\left(\frac{r_i}{R}\right)\sin\varphi_i}\right\}\right)}\right] \qquad [1](8)$$

- Update the rotational induction factor:

$$a' = \frac{1}{\left[4F\frac{\sin\varphi_i\cos\varphi_i}{\sigma_i' C_t}-1\right]} \tag{9}$$

7. Convergence Check:
   - If the induction factors change more than a specified tolerance, repeat from step 2; otherwise, proceed to the next step.
8. Compute the differential thrust and torque of each element and added up to find the total thrust and torque using:

$$dT = \frac{1}{2}B\rho U_{rel}^2 c\,dr\,C_n$$

$$dQ = \frac{1}{2}B\rho U_{rel}^2 c\,r\,dr\,C_t \tag{10}$$

Where:

$$U_{rel} = \sqrt{U_{ax}^2 + U_{th}^2}$$

$$U_{ax} = (1+a)U$$

$$U_{th} = (1-a')r\frac{rpm*2\pi}{60} \tag{11}$$

- To obtain the total thrust (T) and total torque (Q) for the propeller, sum up the differential thrust and torque across all blade elements:

$$\mathrm{T} = \sum_{i=1}^{N} dT_i$$

$$\mathrm{Q} = \sum_{i=1}^{N} dQ_i \tag{12}$$

9. Compute the dimensionless coefficient using:

$$C_T = \frac{T}{r*rps^2*D^4}$$

$$C_Q = \frac{Q}{r*rps^2*D^5}$$

$$C_P = 2\pi * C_Q \tag{13}$$

Where:

$$rps = \frac{rpm}{2\pi} \tag{14}$$

10. Compute the efficiency using:

$$\eta = \frac{T*U}{Power} \tag{15}$$

Where:

$$Power = Q*\Omega \tag{16}$$

Having detailed the algorithm for propeller performance analysis using Blade Element Momentum Theory (BEMT), we now focus on the specific propeller characteristics and operational flying conditions utilized to test and validate our MATLAB code.

Propeller Characteristics and Operational Flying Conditions

The primary aim of this project is to develop MATLAB code specifically designed to analyze propeller performance. For practical application and validation of this code, we have selected the APC 10x7 Thin Electric propeller as our test subject. This propeller was chosen for its commonly used size and easy accessibility, factors that are essential for conducting a robust and detailed analysis. These characteristics not only facilitate a deeper understanding of propeller dynamics but also ensure that our findings have real-world applicability.[3]

Propeller Specifications

Here, we outline the key parameters of the propeller geometry. This detailed information is crucial for accurately simulating and analyzing the propeller's performance across various operational scenarios.

- Propeller Diameter (D): 0.254 meters.
- Number of Blades (B): Two blades.
- Airfoil Shape: NACA 4412 airfoil.

Operational Flight Conditions

The code tests the propeller under specified conditions to simulate realistic operational environments:

- Air Density: The density of the air is set at 1.1991 kg/m³, essential for calculating the air mass interacting with the propeller blades.
- Engine RPM: Fixed at 6519 RPM, the rotational speed is a pivotal factor affecting the airflow dynamics and energy transfer efficiency.
- Free Stream Velocity: Varied between 10 and 23.5 m/s, these velocities help assess the propeller's response under different speeds and dynamic pressures.

The Table 1 outlines the variation of blade element characteristics across the radial length of the APC 10x7 Thin Electric propeller. These parameters are crucial as they influence the aerodynamic forces experienced by each blade segment under operational conditions, which are vital for our analysis using Blade Element Momentum Theory.

Table 1
Blade Element Characteristics

r/R	c/R	Beta(deg)
0.15	0.138	37.86
0.2	0.154	45.82
0.25	0.175	44.19
0.3	0.19	38.35
0.35	0.198	33.64
0.4	0.202	29.9
0.45	0.2	27.02
0.5	0.195	24.67
0.55	0.186	22.62
0.6	0.174	20.88
0.65	0.161	19.36
0.7	0.145	17.98
0.75	0.129	16.74
0.8	0.112	15.79
0.85	0.096	14.64
0.9	0.081	13.86
0.95	0.061	12.72
1	0.04	11.53

Each blade element's characteristics are adjusted based on their position along the blade, affecting how each segment contributes to the overall thrust and torque generated by the propeller. The data in this table allows us to compute the aerodynamic loads and performance for each segment using our MATLAB code, providing a detailed and comprehensive analysis of propeller behavior under various operating conditions.

RESULTS

This section presents the results of the propeller performance analysis using the Blade Element Momentum Theory (BEMT) model implemented in MATLAB. The results include the thrust coefficient, torque coefficient, power coefficient, and propulsive efficiency of the APC 10x7 Thin Electric propeller under various advance ratios.

Thrust Coefficient vs Advance Ratio

The thrust coefficient (CT) as a function of the advance ratio (J) is presented in Figure 1. The graph shows a decreasing trend in the thrust coefficient with an increase in the advance ratio, indicating how the propeller's ability to generate thrust diminishes as the advance ratio increases [1], [4].

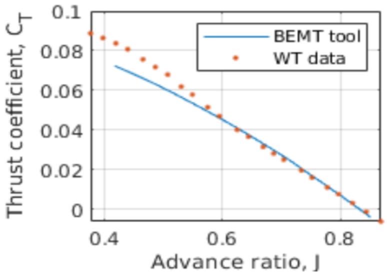

Figure 1
Thrust coefficient vs Advance ratio for APC 10x7 Thin Electric Propeller

Torque Coefficient vs Advance Ratio

Figure 2 depicts the torque coefficient (CQ) versus the advance ratio. A parabolic decline in the torque coefficient is observed with increasing advance ratio, illustrating the relationship between the torque generated by the propeller and its advance ratio.

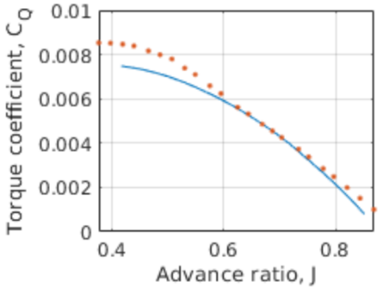

Figure 2
Torque Coefficient vs Advance Ratio for APC 10x7 Thin Electric Propeller

Power coefficient vs Advance Ratio

Figure 3 shows the power coefficient (CP) versus the advance ratio. The power coefficient follows a parabolic decrease, with values ranging from 0.052 at 0.34 advance ratio to 0.008 at an advance ratio of 0.90.

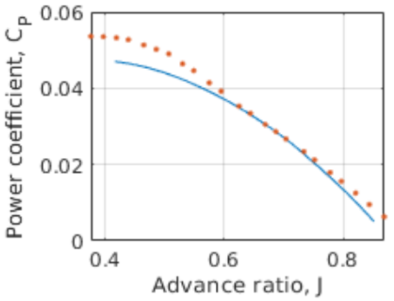

Figure 3
Power coefficient vs Advance ratio for APC 10x7 Thin Electric propeller

Propulsive efficiency vs Advance Ratio

The propulsive efficiency (η) as a function of the advance ratio is illustrated in Figure 4. The propulsive efficiency remains relatively constant between 0.34 and 0.7 advance ratios, maintaining values between 0.65 and 0.75. Beyond an advance ratio of 0.7, a rapid decline is observed, dropping to zero at an advance ratio of 0.85. This rapid decrease highlights the inefficiency of the propeller at higher advance ratios.

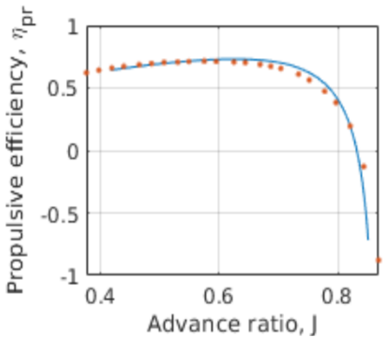

Figure 4
Propulsive Efficiency vs Advance Ratio APC 10x7 Thin Electric Propeller

ANALYSIS OF RESULTS

The APC 10x7 Thin Electric propeller, featuring two blades and a constant NACA 4412 airfoil shape, was analyzed at a fixed rotational speed of 6519 RPM. The MATLAB simulation results were compared with empirical wind tunnel data, focusing on thrust coefficient (CT), torque coefficient (CQ), power coefficient (CP), and propulsive efficiency (η) as functions of the advance ratio (J).

Thrust Coefficient (CT)

The thrust coefficient decreases quasi-linearly with the advance ratio, starting from a maximum of 0.88 at an advance ratio of 0.34 and dropping to -0.1 at 0.90. This trend indicates a reduction in the propeller's thrust-generating capability as the advance ratio increases. The quasi-linear behavior suggests that the propeller becomes less efficient at converting rotational motion into thrust as the forward speed increases relative to the propeller speed [1] [4].

Torque Coefficient (CQ)

The torque coefficient exhibits a parabolic decline, with a maximum value of 0.0083 at an advance ratio of 0.34 and a minimum of 0.001 at 0.90. This parabolic trend highlights the diminishing torque generated by the propeller as the advance ratio increases. The slope of the decline steepens with increasing advance ratio, indicating that higher advance ratios significantly reduce the torque.

Power Coefficient (CP)

Similarly, the power coefficient follows a parabolic decrease, with values ranging from 0.052 at an advance ratio of 0.34 to 0.008 at 0.90. The power coefficient's behavior reflects the combined effects of decreasing thrust and torque as the advance ratio increases, illustrating the propeller's reduced ability to convert engine power into useful work at higher speeds.

Propulsive Efficiency (η)

The propulsive efficiency remains relatively constant between advance ratios of 0.34 and 0.70, maintaining values between 0.65 and 0.75. However, beyond an advance ratio of 0.70, there is a rapid decline, with efficiency dropping to zero at an advance ratio of 0.85. This rapid decrease emphasizes the propeller's inefficiency at higher advance ratios, where the forward speed becomes too great relative to the rotational speed, leading to a significant loss in efficiency.

Model Accuracy and Limitations

Overall, the results indicate that the proposed BEMT model provides accurate predictions for propeller performance at advance ratios greater than 0.40. However, the model's accuracy diminishes at lower advance ratios. This discrepancy may be attributed to the limitations of the BEMT model in accounting for complex flow phenomena such as three-dimensional effects and viscous interactions, which become more pronounced at lower advance ratios.

CONCLUSION

This study successfully developed a MATLAB-based tool for analyzing propeller performance using Blade Element Momentum Theory (BEMT). The tool was validated using the APC 10x7 Thin Electric propeller, demonstrating its effectiveness in predicting performance metrics such as thrust coefficient (CT), torque coefficient (CQ), power coefficient (CP), and propulsive efficiency (η) across various advance ratios.

Key insights from the study include:

- Thrust, Torque, and Power Coefficients: All decrease with increasing advance ratio, indicating reduced performance at higher speeds. [1,4]
- Propulsive Efficiency: Remains stable up to a certain point but drops sharply beyond an advance ratio of 0.70, highlighting the importance of matching propeller design to operational conditions. [1,4]
- Model Reliability: The BEMT model is reliable for higher advance ratios but requires refinement or additional corrections for low advance ratios to enhance accuracy.

Future work should focus on enhancing the model to include three-dimensional effects and viscous interactions, thereby improving its accuracy across a broader range of conditions. This study provides a solid foundation for future research and development in propeller analysis and optimization, with potential applications in aerospace, marine, and wind energy sectors.

Overall, the developed MATLAB code bridges the gap between theoretical assumptions and practical applications, contributing to more efficient and effective propeller designs.

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